

The Self-Interacting Curvaton

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The evolution of the curvature perturbation is highly non-trivial for curvaton models with self-interactions and is very sensitive to the parameter values. The final perturbation depends also on the curvaton decay rate Γ . As a consequence, non-gaussianities can be greatly different from the purely quadratic case, even if the deviation is very small. Here we consider a class of polynomial curvaton potentials and discuss the dynamical behavior of the curvature perturbation. We point out that, for example, it is possible that the non-gaussianity parameter $f_{\text{NL}} \simeq 0$ while g_{NL} is non-zero. In the case of a curvaton with mass $m \sim \mathcal{O}(1)$ TeV we show that one cannot ignore non-quadratic terms in the potential, and that only a self-interaction of the type $V_{\text{int}} = \sigma^8/M^4$ is consistent with various theoretical and observational constraints. Moreover, the curvaton decay rate should then be in the range $\Gamma = 10^{-15} - 10^{-17}$ GeV.

§1. Introduction

In the curvaton mechanism,¹⁾ primordial perturbations originate from quantum fluctuations of a light scalar field which gives a negligible contribution to the total energy density during inflation. This field is called the curvaton σ . Inflation is driven by another scalar, the inflaton ϕ , whose potential energy dominates the universe. After the end of inflation, the inflaton decays into radiation. If the inflationary scale is low enough, $H_* \ll 10^{-5} \sqrt{\epsilon_*}$, the density fluctuations of the radiation component are much below the observed amplitude $\delta\rho/\rho \sim 10^{-5}$ and the fluid is for practical purposes homogeneous. While the dominant radiation energy scales away as $\rho_r \sim a^{-4}$, the curvaton contribution to the total energy density may increase and the initially negligible curvaton perturbations get imprinted into the metric. The standard adiabatic hot big bang era is recovered when the curvaton eventually decays and thermalizes with the existing radiation. The mechanism can be seen as a conversion of initial isocurvature perturbations into adiabatic ones and, depending on the parameters of the model, is capable of generating all of the observed primordial perturbation. The scenario sketched above represents the simplest possible realization of the curvaton mechanism, and a wide range of different variations of the idea have been studied in the literature. For example, the inflaton perturbations need not be negligible,²⁾ there could be several curvatons,³⁾ the curvaton decay can result into residual isocurvature perturbations^{4),5)} and inflation could be driven by some other mechanism than slowly rolling scalars.⁶⁾

It is however well known that the predictions of the curvaton model are quite sensitive to the form of the curvaton potential.^{7)–17)} In particular, even small deviations from the extensively studied quadratic potential can have a significant effect, at least when considering non-Gaussian effects.^{8),9)} One can also encounter strong scale-dependence of the non-gaussianity parameters.¹⁸⁾ When the initial curvaton

field value lies far in the non-quadratic part of the potential, the non-linear nature of the evolution equation will in general result in a very rich structure of phenomena in the parameter space, as has been discussed in detail in 14) and 15).

The simplest non-quadratic curvaton potential is given by

$$V = \frac{1}{2}m^2\sigma^2 + \lambda\frac{\sigma^{n+4}}{M^n}, \quad (1.1)$$

where n is an even integer to keep the potential bounded from below, and the interaction term is suppressed by a cut-off scale M . For non-renormalizable operators $n > 0$, we set the cut-off scale to be the Planck scale $M = M_{\text{P}} \equiv 1$, and the coupling to unity, $\lambda = 1$. For the renormalizable quartic case $n = 0$, the coupling λ can be treated as a free parameter.

The potential (1.1) is reasonably well motivated by generic theoretical arguments. Indeed, the curvaton should have interactions of some kind as it eventually must decay and produce Standard Model fields. The curvaton needs to be weakly interacting to keep the field light during inflation. This however only implies that the effective curvaton potential should be sufficiently flat in the vicinity of the field expectation value during inflation but does not a priori require the interaction terms in (1.1) to be negligible. Moreover, as typically the inflationary energy scale is relatively high, the field can be displaced far from the origin and therefore feels the presence of the higher order terms in the potential. The interactions could arise either as pure curvaton self-interactions involving the curvaton field σ alone, or more generically as effective terms due to curvaton couplings to other (heavy) degrees of freedom that have been integrated out. An example of a possible physical setup which could lead to (1.1) is given by flat directions of supersymmetric models that have been suggested as curvaton candidates.¹⁹⁾ These would lead to a potential of the form (1.1) with typically a relatively large power for the non-renormalizable operator.

The amplitude and non-gaussianity of the perturbation depend on the curvaton decay time. Here we assume for simplicity a perturbative curvaton decay characterized by some effective decay width Γ (for non-perturbative decay, see 20) and 21)).

When the interaction term dominates in (1.1), the curvaton oscillations start in a non-quadratic potential and the curvaton energy density always decreases faster than for a quadratic case. For non-renormalizable interactions, the decrease is even faster than the red-shifting of the background radiation and the curvaton contribution to the total energy density is decreasing at the beginning of oscillations. Consequently, the amplification of the curvaton component is less efficient than for a quadratic model. For the same values for m and Γ , the curvaton typically ends up being more subdominant at the time of its decay than in the quadratic case.

Despite the subdominance, the curvaton scenario can yield the correct amplitude of primordial perturbations as the relative curvaton perturbation $\delta\sigma_*/\sigma_*$ produced during inflation can be much larger than 10^{-5} . For a quadratic model, it is well known that the curvaton should make up at least few per cents of the total energy density at the time of its decay in order not to generate too large non-

gaussianities.^{4),22)} This bound does not directly apply to the non-quadratic model (1.1) since the dynamics is much more complicated. Although the subdominant curvaton scenario implies relatively large perturbation $\delta\sigma_*/\sigma_*$, the higher order terms in the perturbative expansion of curvature perturbation can be accidentally suppressed.^{8),9),15)}

§2. The curvature perturbation

Let us adopt the δN formalism^{23),24)} and assume that the curvature perturbation ζ arises solely from the inflation generated perturbation of a single curvaton field. Then

$$\zeta(t, \mathbf{x}) = N'(t, t_*)\delta\sigma_*(\mathbf{x}) + \frac{1}{2}N''(t, t_*)\delta\sigma_*(\mathbf{x})^2 + \frac{1}{6}N'''(t, t_*)\delta\sigma_*(\mathbf{x})^3 \dots \quad (2.1)$$

Here $N(t, t_*)$ is the number of e-foldings from an initial spatially flat hypersurface with fixed scale factor $a(t_*)$ to a final hypersurface with fixed energy density $\rho(t)$, evaluated using the FRW background equations. The final time t is some arbitrary time after the curvaton decay. The prime denotes a derivative with respect to the initial curvaton value σ_* . Here we take t_* to be some time during inflation soon after all the cosmologically relevant modes have exited the horizon and assume that the curvaton perturbations $\delta\sigma_*$ are Gaussian at this time. The expansion (2.1) is then of the form

$$\zeta(t, \mathbf{x}) = \zeta_g(t, \mathbf{x}) + \frac{3}{5}f_{\text{NL}}\zeta_g(t, \mathbf{x})^2 + \frac{9}{25}g_{\text{NL}}\zeta_g(t, \mathbf{x})^3 + \dots \quad (2.2)$$

where $\zeta_g(t, \mathbf{x})$ is a Gaussian field and the non-linearity parameters are given by

$$f_{\text{NL}} = \frac{5}{6} \frac{N''}{N'^2}, \quad (2.3)$$

$$g_{\text{NL}} = \frac{25}{54} \frac{N'''}{N'^3}. \quad (2.4)$$

Here we neglect all the scale dependence of the non-linearity parameters.^{18),25)} With this assumption, and neglecting higher order perturbative corrections, the constants f_{NL} and g_{NL} measure the amplitudes of the three- and four-point correlators of ζ , respectively. Observationally, they can be extracted from the CMB bi- and trispectra.

We assume the curvaton obeys slow roll dynamics during inflation and introduce a parameter r_* to measure its contribution to the total energy density ρ at t_* :

$$r_* = \left. \frac{\rho_\sigma}{\rho} \right|_{t_*} \simeq \frac{V(\sigma_*)}{3H_*^2} \ll 1. \quad (2.5)$$

Here the inflationary scale H_* is a free parameter, up to certain model dependent consistency conditions. Assuming inflation is driven by a slowly rolling inflaton field, we need to require $H_* \ll 10^{-5}\sqrt{\epsilon}$ in order to make the inflaton contribution to ζ negligible. In this setup we also need to adjust the slow-roll parameter $\epsilon = -\dot{H}_*/H_*^2$,

determined by the inflaton dynamics, to give the correct spectral index,²⁶⁾ $n - 1 = 2\epsilon - 2\eta_{\sigma\sigma}$. The curvaton contribution, $\eta_{\sigma\sigma} = V''(\sigma_*)/3H_*^2$, is typically negligible because of the subdominance of the curvaton. The curvaton mass is required to be small, but the same also holds for the inflaton mass.

After the end of inflation, one assumes that the inflaton decays completely into radiation and the universe becomes radiation dominated. The decay constant Γ accounts for the coupling between the radiation and the curvaton component. The evolution of the coupled system is then given by

$$\ddot{\sigma} + (3H + \Gamma)\dot{\sigma} + m^2\sigma + \lambda(n + 4)\sigma^{n+3} = 0, \quad (2.6)$$

$$\dot{\rho}_r = -4H\rho_r + \Gamma\dot{\sigma}^2, \quad (2.7)$$

$$3H^2 = \rho_r + \rho_\sigma. \quad (2.8)$$

The initial conditions are given by $\rho_r = 3H_*^2$ and $\rho_\sigma = V(\sigma_*) = r_*/(1 - r_*)\rho_r$ specified at time t_* corresponding to the end of inflation. We also set $\dot{\sigma} = 0$. Given the parameters n , λ and m , which determine the potential (1.1), and the two initial conditions r_* and H_* , one can calculate N in (2.1) from this set of equations. To find the curvature perturbation, we set $\delta\sigma_* = H_*/(2\pi)$ and compute $\zeta = N(\sigma_* + \delta\sigma_*) - N(\sigma_*)$. For a given set of parameters, one then adjusts the decay width Γ so that the observed amplitude²⁷⁾ $\zeta \sim 10^{-5}$ is obtained.

One may treat Γ as a free parameter since we have not specified the curvaton couplings to other matter, in particular to the Standard Model fields. However, since the primordial perturbation is mostly adiabatic, the curvaton should decay before dark matter decouples in order not to produce isocurvature modes. Assuming a freeze-out temperature $T \simeq m_{\text{DM}}/20$ for an LSP type dark matter model with the LSP mass $m_{\text{DM}} \sim \mathcal{O}(100)$ GeV, this translates to a rough bound

$$\Gamma \gtrsim 10^{-17} \text{GeV}. \quad (2.9)$$

While this bound could be relaxed in non-minimal constructions, let us here adopt (2.9) for definiteness.

§3. Small deviations from the quadratic form

Let us first assume that the deviation from the quadratic form is small and write the curvaton potential (1.1) as^{8),9)}

$$V(\sigma) = \frac{1}{2}m^2\sigma^2 + \lambda m^4 \left(\frac{\sigma}{m}\right)^n, \quad (3.1)$$

where λ is some coupling constant. It is also useful to define a parameter s which represents the size of the non-quadratic term relative to the quadratic one:

$$s \equiv 2\lambda \left(\frac{\sigma_*}{m}\right)^{n-2}. \quad (3.2)$$

Thus the larger s is, the larger is the contribution from the non-quadratic term.

The curvature fluctuation can then be written, up to the third order, as²⁸⁾

$$\begin{aligned} \zeta = \delta N = & \frac{2}{3} r \frac{\sigma'_{\text{osc}}}{\sigma_{\text{osc}}} \delta\sigma_* + \frac{1}{9} \left[3r \left(1 + \frac{\sigma_{\text{osc}} \sigma''_{\text{osc}}}{\sigma'^2_{\text{osc}}} \right) - 4r^2 - 2r^3 \right] \left(\frac{\sigma'_{\text{osc}}}{\sigma_{\text{osc}}} \right)^2 (\delta\sigma_*)^2 \\ & + \frac{4}{81} \left[\frac{9r}{4} \left(\frac{\sigma_{\text{osc}}^2 \sigma'''_{\text{osc}}}{\sigma'^3_{\text{osc}}} + 3 \frac{\sigma_{\text{osc}} \sigma''_{\text{osc}}}{\sigma'^2_{\text{osc}}} \right) - 9r^2 \left(1 + \frac{\sigma_{\text{osc}} \sigma''_{\text{osc}}}{\sigma'^2_{\text{osc}}} \right) \right. \\ & \left. + \frac{r^3}{2} \left(1 - 9 \frac{\sigma_{\text{osc}} \sigma''_{\text{osc}}}{\sigma'^2_{\text{osc}}} \right) + 10r^4 + 3r^5 \right] \left(\frac{\sigma'_{\text{osc}}}{\sigma_{\text{osc}}} \right)^3 (\delta\sigma_*)^3, \end{aligned} \quad (3.3)$$

where σ_{osc} is the value of the curvaton at the onset of its oscillation; above^{*})

$$r \equiv \frac{3\rho_\sigma}{4\rho_{\text{rad}} + 3\rho_\sigma} \Big|_{\text{decay}}. \quad (3.4)$$

Notice that $\sigma'_{\text{osc}}/\sigma_{\text{osc}} = 1/\sigma_*$ for the case of the quadratic potential. With this expression, we can write down the non-linearity parameter f_{NL} as

$$f_{\text{NL}} = \frac{5}{4r} (1 + S) - \frac{5}{3} - \frac{5r}{6}, \quad (3.5)$$

where we have defined

$$S = \frac{\sigma_{\text{osc}} \sigma''_{\text{osc}}}{\sigma'^2_{\text{osc}}} \quad (3.6)$$

with $S = 0$ for a purely quadratic potential. Also notice that, although the curvaton scenario generally generates large non-gaussianity with $f_{\text{NL}} \gtrsim \mathcal{O}(1)$, the non-linearity parameter f_{NL} can be very small in the presence of the non-linear evolution of the curvaton field,^{8),28)} which can render the term $1 + S \simeq 0$.

The non-linearity parameter g_{NL} associated with the trispectrum can be written as

$$g_{\text{NL}} = \frac{25}{54} \left[\frac{9}{4r^2} \left(\frac{\sigma_{\text{osc}}^2 \sigma'''_{\text{osc}}}{\sigma'^3_{\text{osc}}} + 3S \right) - \frac{9}{r} (1 + S) + \frac{1}{2} (1 - 9S) + 10r + 3r^2 \right]. \quad (3.7)$$

As one can easily see, even if the non-linear evolution of σ cancels to give a very small f_{NL} , such a cancellation does not necessarily occur for g_{NL} . Examples⁹⁾ of the behavior of the non-linearity parameters for various types of self-interaction are depicted in Fig. 1.

§4. General results

Let us now relax the assumption that the deviations from the quadratic case are small. The numerical solutions of the equations of motion (2.6) exhibit complicated behaviour not qualitatively present in the simplified analytical approximation. To demonstrate this, in Fig. 2 we plot $\zeta = \Delta N$ as a function of $H(t)$, or the inverse of

^{*}) Note that in the literature there is much variation in the definition of r , as is also in the present paper.

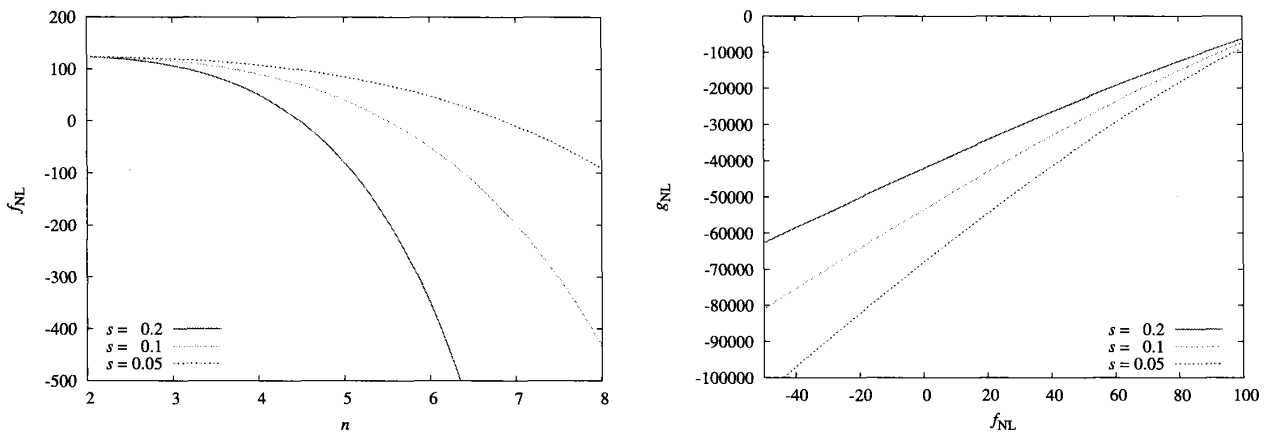


Fig. 1. (Left) Plot of f_{NL} as a function of n for several values of s . (Right) Plot of g_{NL} as a function of f_{NL} for several values of s . Notice that f_{NL} and n have one-to-one correspondence. In both panels, $r = 0.01$.

time, for fixed initial values of H_* , r_* (see (2.5)) and Γ for two different curvaton mass values. For different masses the moment of transition from the non-renormalizable part of the potential to the quadratic part of the potential is different, and this affects strongly the final value of ΔN which is dictated by the duration of oscillations in the non-quadratic regime. Deep in the quadratic regime the oscillations become faster and faster so that ΔN 's evolution is given by a scaling law.¹⁴⁾ However, before the quadratic term in the potential starts to dominate, ΔN shows complicated oscillatory behaviour that can be only tracked numerically.

From Fig. 2 it is clear that as the field value oscillates in time, so does ΔN . In the non-quadratic regime ΔN oscillates with a large amplitude. If the transition to the quadratic regime is slow compared to the oscillations in the non-quadratic regime, the transition averages over several oscillations. As a consequence, the final value of ΔN will be a non-oscillatory function of the model parameters. However, if the oscillation frequency in the non-quadratic potential is slow, and the transition to the quadratic oscillations is rapid, then the phase of the non-quadratic oscillation affects the final value of ΔN . If the parameters happen to be such that the transition to the quadratic regime occurs at a maximum of the oscillation, a relatively high value of ΔN freezes out. Similarly, if the transition occurs at a minimum of the oscillation cycle, the final value of ΔN will be much smaller. If the parameters governing the moment of transition, such as the curvaton mass m , are changed continuously, then the phase of the non-quadratic oscillation during the transition also changes continuously. In the space of the parameters this results in an oscillatory pattern in ΔN . This behaviour can be understood by observing that the curvaton energy density at the beginning of its oscillations in the non-quadratic part of the potential can not be expressed in terms of an amplitude of the envelope alone but also depends on the phase of the oscillation, or equivalently on both the field σ and its time derivative $\dot{\sigma}$, in a non-trivial way. In effect, these act as two independent dynamical degrees of freedom. If the transition from the interaction dominated part to the quadratic region takes place at this stage, the initial variation of the curvaton

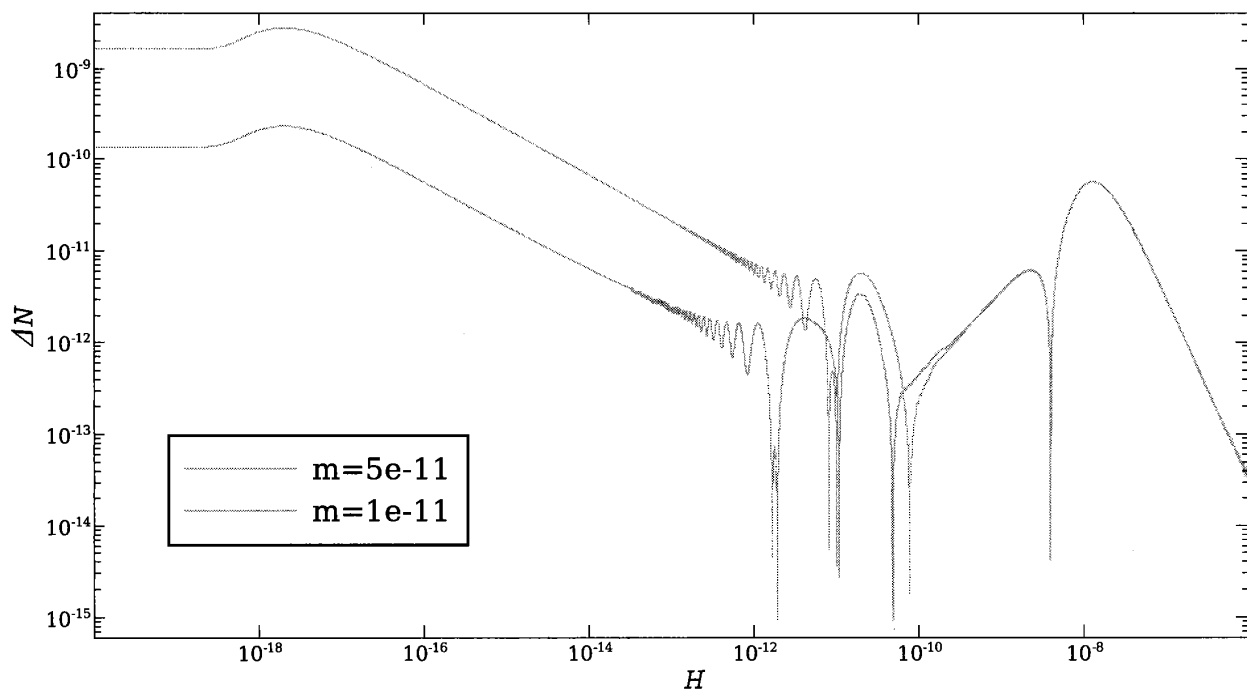


Fig. 2. $\zeta = \Delta N$ as a function of H , (time evolves from right to left), for the masses $m = 10^{-11}$ (lower curve) and $m = 5 \cdot 10^{-11}$ (upper curve). In both examples $H_* = 10^{-6}$, $n = 4$, $\Gamma = 10^{-18}$ and $r_* = 10^{-10}$.

value σ_* can therefore translate in a non-trivial fashion into the final value of the curvature perturbation.

For a potential with $V \sim \sigma^{n+4}$ no oscillatory solutions exist if $n \geq 6$. This means that in the non-quadratic regime the curvaton merely decays and hence no oscillations in ΔN occur. In 14) and 15) we have scanned the parameter space (m, n, Γ) to find the regions that yield $\zeta \simeq 10^{-5}$ and acceptably small non-gaussianity while being consistent with the slow-roll assumption. For fixed parameter values, the result can be mapped out as a region in the space of the inflation scale H_* and the initial relative curvaton energy fraction, r_* . We also bound H_* from above by $H_* \lesssim 10^{-5}$, in order to prevent the excessive production of primordial tensor modes and to keep the inflaton perturbation negligible.

The experimental limits for f_{NL} are given by the WMAP 5-year data,²⁷⁾ $-9 < f_{\text{NL}} < 111$; we also require that $-3.5 \times 10^5 < g_{\text{NL}} < 8.2 \times 10^5$ as given in 29). The schematic outcome of the scan is depicted in Fig. 3.

The observational limits for f_{NL} and g_{NL} constrain the allowed area in the very subdominant regions of the parameter space, depicted in Fig. 3 by the line *a*. Other constraints shown in Fig. 3 arise from the internal consistency of the self-interacting curvaton scenario. The bound *b* is obtained because otherwise the initial perturbations would be too small to produce the observed amplitude. The bound *c* reflects the requirement that the curvaton should be massless, or $V'' < H_*^2$, which is necessary for the generation of curvaton perturbations during inflation. Because of the subdominance of the curvaton, the realistic bound should arguably be a few orders of magnitude tighter. However, a change of an order of magnitude moves the actual

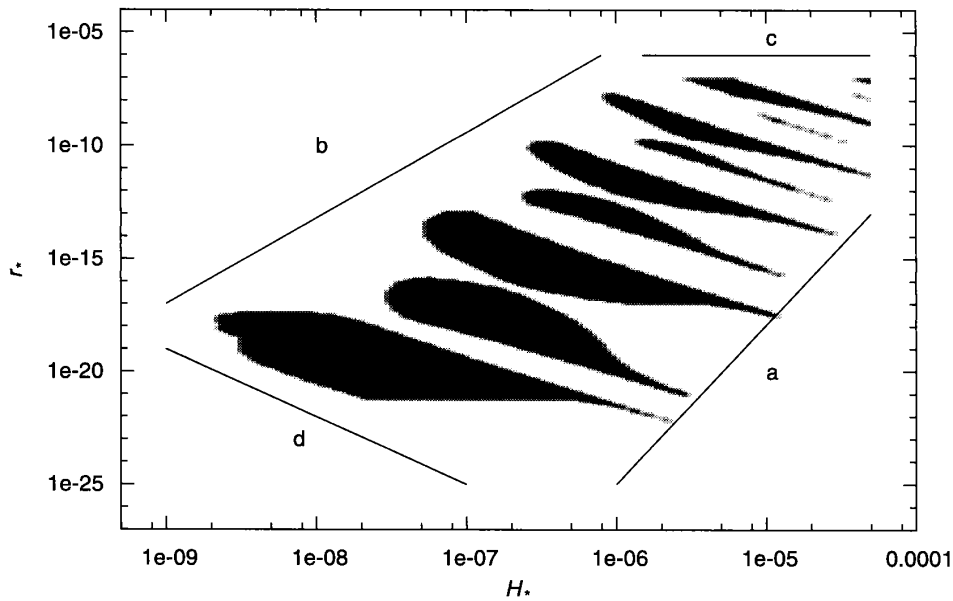


Fig. 3. A schematic illustration of the different cuts limiting the allowed area in the parameter space of the inflation scale H_* and the initial curvaton energy density relative to the total density r_* .

cut by a very small amount in the log-log plots. Finally, the bound d guarantees the absence of the isocurvature modes in dark matter perturbations and corresponds to the limit on the curvaton decay width given in (2.9). Detailed figures that show the allowed regions of H_* and r_* for different parameter values can be found in 15).

§5. TeV mass curvaton

It is of particular interest to consider the self-interacting curvaton with a mass $m \simeq 1$ TeV. Examples of such a curvaton could be found among the MSSM flat directions³⁰⁾ or light moduli fields of string theories. Let us therefore fix $m = 1$ TeV and demonstrate first that a simple non-interacting or quadratic form does not give rise to a consistent curvaton model, as discussed in 17).

For a quadratic curvaton potential one can write the perturbation as

$$\zeta \sim \frac{H_*}{\sigma_*} r_{\text{eff}} \simeq 10^{-5}, \tag{5.1}$$

where H_*/σ_* gives the initial perturbation amplitude in the curvaton, and r_{eff} is the efficiency factor that can be approximated quite well by the energy fraction at the curvaton decay:⁴⁾

$$r_{\text{eff}} \approx r_{\text{dec}} \equiv \left. \frac{\rho_\sigma}{\rho_r + \rho_\sigma} \right|_{\text{decay}}. \tag{5.2}$$

Relating σ_* and r_* from $\frac{1}{2}m^2\sigma_*^2/3M_{\text{Pl}}^2H_*^2 \simeq r_*$, and noting that $r_{\text{dec}} < 1$, we find the

constraint on the initial curvaton energy fraction

$$r_* < \frac{1}{6} \frac{m^2}{\zeta^2 M_{\text{Pl}}^2}. \quad (5.3)$$

In the free curvaton case r_{dec} also determines non-gaussianity through the simple relation⁴⁾ $f_{\text{NL}} = 5/4 r_{\text{dec}}$. Very roughly, observationally $|f_{\text{NL}}| < 100$, which implies the constraint

$$r_* > \frac{10^{-4}}{6} \frac{m^2}{\zeta^2 M_{\text{Pl}}^2}. \quad (5.4)$$

The limits (5.3) and (5.4) are well known. However, there is more. Since the observed perturbations are adiabatic to great accuracy, the curvaton must decay before dark matter decouples. For each set of the initial conditions, (H_*, r_*) , there is a relation between r_{dec} in (5.2) and the effective decay constant Γ given by the fact that decay time is defined as $H = \Gamma$. Here we assume implicitly a perturbative curvaton decay, but Γ could stand for any effective inverse decay time and thus the following discussion should hold, at least roughly, also for a non-perturbative curvaton decay as discussed in 20) (note however that non-perturbative curvaton decay could turn out to be a source of a considerable non-gaussianity¹⁶⁾).

The exact evolution of the energy densities is difficult to solve analytically. However, we can approximate the curvaton evolution by dividing it up to three phases:

1. When $V'' = m^2 < H^2$, the curvaton is effectively massless, so the field value stays constant, $\sigma = \sigma_*$.
2. When $V'' = m^2 > H^2$, the curvaton oscillates in the quadratic potential, and thus its energy density approximately scales as $\rho_\sigma \propto a^{-3}$.
3. The curvaton oscillates until $H = \Gamma$, whence it decays.

Solving the Friedmann equation for the regime where $m^2 > H^2$ then yields

$$\frac{a(H)}{a_*} = \sqrt{\frac{H_*}{H}} \left\{ 1 + \frac{r_*}{4} \left[\frac{H_*^2}{m\sqrt{H_*m}} - 1 \right] \right\} + \mathcal{O}(r_*^2).$$

Using the above result we can solve for r_* to find

$$r_* = \frac{m\sqrt{m\Gamma}}{H_*^2} \frac{6 \left(\frac{M_{\text{Pl}}}{m} \right)^2 \zeta^2}{\frac{H_*^2}{m\sqrt{m\Gamma}} - 12 \left(\frac{M_{\text{Pl}}}{m} \right)^2 \zeta^2}. \quad (5.5)$$

We need to check whether, given the constraints discussed above, the self-interactions can be neglected if $m \simeq 1$ TeV. Thus, adopting the form of the potential given in (1.1), in order for the quadratic assumption to be consistent, we should require that

$$\frac{1}{2} m^2 \sigma^2 \gg \frac{\sigma^{n+4}}{M_{\text{Pl}}^n} \quad (5.6)$$

throughout the evolution. Since the energy density of the quadratic field decreases monotonously, it is sufficient to apply this requirement only for the initial conditions.

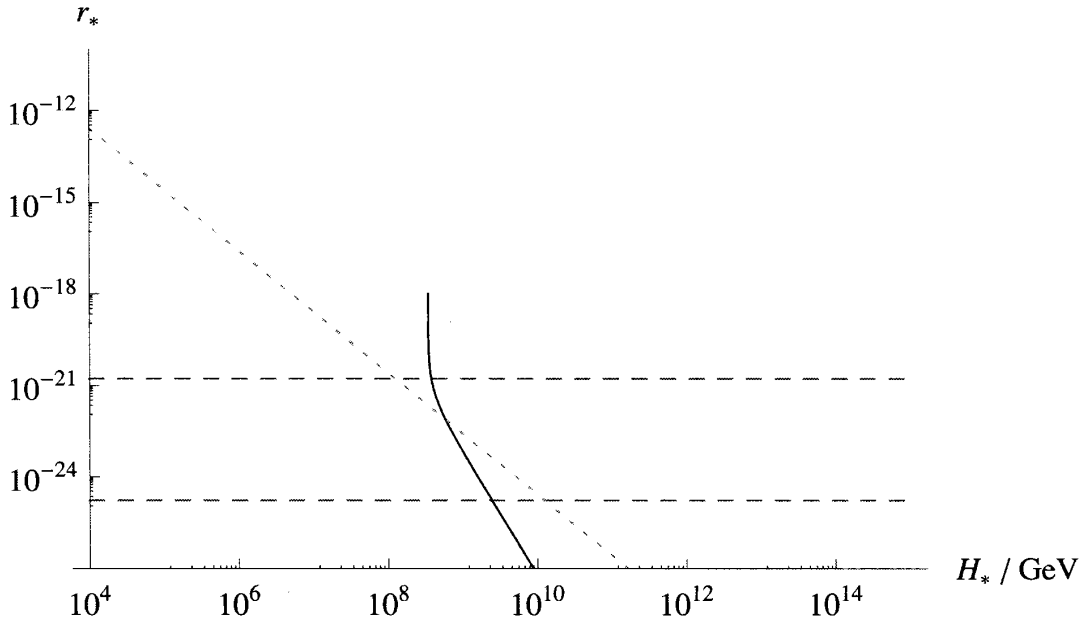


Fig. 4. (Color online) Parameter space of the quadratic curvaton. r_* must be above the lower horizontal dashed line to produce $\zeta \sim 10^{-5}$ (Eq. (5.3)) and below the blue upper horizontal dashed line to produce small enough f_{NL} (Eq. (5.4)). Furthermore, Γ is constrained from above, and thus only the parameter space to the right of the black solid line is allowed (Eq. (5.5)). The green dotted line illustrates the equality of the mass term and a possible self-interaction term in the potential (Eq. (5.7)) for $n = 4$. For smaller values of n the line moves further to the left. To the right of the dotted line the self-interaction dominates, and thus practically in all of the allowed parameter space the self-interaction must be taken into account.

Solving for r_* such that the magnitudes of the quadratic and non-quadratic terms are equal, we find the condition

$$r_* = \frac{m^2}{3M_{\text{Pl}}^2 H_*^2} \left(\frac{m^2 M_{\text{Pl}}^n}{2} \right)^{\frac{2}{n+2}}. \quad (5.7)$$

We have plotted this condition for $n = 4$ in Fig. 4 as the diagonal dotted line. To the right of it, the non-quadratic term dominates initially. As can be seen in Fig. 4, there is practically no allowed region in the parameter space where the quadratic assumption would even approximately apply. For smaller values of n , the self-interaction becomes important even for much smaller values of H_* and r_* , and thus, there is no quadratic regime left in the parameter space.

We thus may conclude that even if the curvaton self-interactions were very weak, a purely quadratic potential would not be a consistent approximation for a mass $m \simeq 1$ TeV; instead, the effects of the self-interactions need to be taken into account. These change the dynamics of the curvaton in a significant way. Moreover, as discussed in 17), a scan of the parameter space reveals that only $n = 4$ potential with $V \sim \sigma^8$ has any allowed parameter space. In addition, in order to obtain a correct perturbation amplitude, the decay width Γ should be in the range $10^{-15} - 10^{-17}$ GeV. For most particle physics models, this would be a rather small decay width. We estimate¹⁷⁾ roughly that in the MSSM, where the non-zero curvaton background

provides masses to other particles and hence gives rise to a kinematical blocking,³¹⁾ one could obtain widths of the order $\Gamma \sim 10^{-12}$. However, a detailed and more proper calculation is required to settle the issue.

§6. Discussion

It may appear surprising that even very small deviations from the quadratic form of the curvaton potential can affect the curvature perturbation in a significant way. However, one should bear in mind that the small curvature perturbation is really the difference of two large numbers. The number of e-folds generated during curvaton oscillations is typically $N \sim \mathcal{O}(10)$, whereas the difference that gives rise to the non-gaussianity is $\Delta N \lesssim 10^{-8}$. Since self-interactions imply non-linearities in the evolution of the curvaton field and in the number of e-folds N , one can understand that even small changes can have profound effects in the difference ΔN . In particular, as discussed here, the non-gaussianities turn out to be quite different as compared with the simplest quadratic model. There the magnitude of f_{NL} in the limit $r_{\text{dec}} \ll 1$ is determined by the curvaton energy density at the time of its decay, $f_{\text{NL}} \sim 1/r_{\text{dec}}$. However, with self-interactions the prediction for f_{NL} can significantly deviate from this simple estimate. Even if $r_{\text{dec}} \ll 1$, there exists regions in the parameter space with $|f_{\text{NL}}| < \mathcal{O}(1)$. This is because the value of f_{NL} oscillates and changes its sign. Nevertheless, g_{NL} can then be very large and one has a rather non-trivial non-Gaussian statistics characterized by a large trispectrum and a vanishing bispectrum. Such a situation, discussed already in 9), appears to be rather generic in self-interacting curvaton models, and is possible for a wide, albeit restricted, range of model parameters. Large non-gaussianities can be generated even if the curvaton dominates the energy density at the time of its decay. In general, in the presence of self-interactions the relative signs of f_{NL} and g_{NL} and the functional relation between them are typically modified from the quadratic case. Thus the non-linearity parameters taken together, in possible conjunction of other cosmological observables such as tensor perturbations, may offer the best prospects for constraining the physical properties of the curvaton.

A TeV mass curvaton is a rather special case. An important constraint, valid also for higher mass curvatons, is that it has to decay before the CDM freeze-out. This, together with observational constraints, fixes the range of the initial conditions for the curvaton field which turn out to be such that the quadratic term in the curvaton potential cannot dominate over possible higher-order terms for the whole dynamical range. One finds¹⁷⁾ that the only viable curvaton potential that satisfies all the constraints is $V = m^2\sigma^2/2 + \sigma^8/M^4$. Moreover, the curvaton decay rate should be in the range $\Gamma = 10^{-15} - 10^{-17}$ GeV. Note that in the case where the curvaton energy density is subdominant at the time of decay, the curvaton does not necessarily have to decay before baryogenesis, which can be a process that takes place among the inflaton decay products. However, the decay should be able to produce thermal CDM particles so that the CDM perturbation is adiabatic.

Note also that what really matters is the equation of state, not the time of decay. Thus if the curvaton decays too early, the perturbations might be still generated if

the decay products have the equation of state of matter. An example of this could be the MSSM flat direction fragmenting into Q-balls, which would then slowly decay.

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